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Common final exam for Math 118, December 15, 2021.

YOUR NAME: _____ SECTION: _____

INSTRUCTOR: _____

DID YOU HAVE ANOTHER EXAM 5:30-7:30 TODAY? _____

Directions:

- Print your name, section number and your instructor's name on this page in the space provided.
- This exam has 12 questions. Please check that your exam is complete.
- You have two hours to complete this exam. It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator and the list of equations provided by the Department.
- When using decimals round your answers till three decimal places.
- Use of notes, books, any internet resources and electronic devices is NOT allowed.
- You may not communicate with anyone besides the instructor during this exam.

Problem	Score
1	/12
2	/9
3	/8
4	/6
5	/6
6	/12
7	/8
8	/6
9	/8
10	/8
11	/5
12	/12

Good luck!

1. (Points: 12) The number of asthma sufferers in the world was about 84 million in 1990 and 334 million in 2012. Let N represent the number of asthma sufferers (in millions) worldwide t years after 1990.

   (a) Model N as a linear function of year t after 1990.
   Answer (3 points): $N = 84 + 11.364 \cdot t$, slope= 11.364, between 1990 and 2012 the number of asthma sufferers is increasing in average by 11.364 million people every year.

   (b) Model N as an exponential function of year t after 1990.
   Answer (3 points): $N = 84 \cdot (1.065)^t$, growth factor= 1.065, between 1990 and 2012 the number of asthma sufferers is increasing by 6.5% every year.

   (c) How many asthma sufferers are predicted worldwide in 2020 with the linear model?
   Answer (3 points): 424.909 million people.

   (d) How many asthma sufferers are predicted worldwide in 2020 with the exponential model?
   Answer (3 points): 551.740 million people.

2. (Points: 9) Rank the following three bank-deposit options from best to worst.

   (a) Bank A: nominal rate 2% compounded daily

   (b) Bank B: nominal rate 2.1% compounded monthly

   (c) Bank C: nominal rate 2.05% compounded continuously

   Answer (3 points for each part): Bank A: APY=2.020%, Bank B: APY=2.120%, Bank C: APY=2.071%. Bank B is the best option, then Bank C, then bank A.

3. (Points: 8) Technetium-99m is a radioactive substance used to diagnose brain diseases. Its half-life is approximately 6 hours. Initially you have 200 mg of technetium-99m.

   (a) Write an equation that gives the amount of the substance remaining after t hours.
   Answer (4 points): $Q(t) = 200 \cdot (0.891)^t$.

   (b) Determine the number of hours needed for your sample to decay to 120 mg.
   Answer (4 points): $t = 4.422$ hours.

4. (Points: 6) What is the long-run behavior of the function given below?

   (a) $x \to \infty, \quad y = \dfrac{x(x+6)(x-9)}{4+x^2} \longrightarrow$

   Answer (3 points):

   $y = \dfrac{x(x+6)(x-9)}{4+x^2} \longrightarrow \infty$

   (b) $x \to -\infty, \quad y = \dfrac{x(x+6)(x-9)}{4+x^2} \longrightarrow$

   Answer (3 points):

   $y = \dfrac{x(x+6)(x-9)}{4+x^2} \longrightarrow -\infty$

5. (Points: 6)

(a) Find the angle between 0° and 360° (but not 240°) that has the same cosine as 240°.

Answer (3 points): 120°.

(b) Find the angle between 0° and 360° (but not 240°) that has the same sine as 240°.

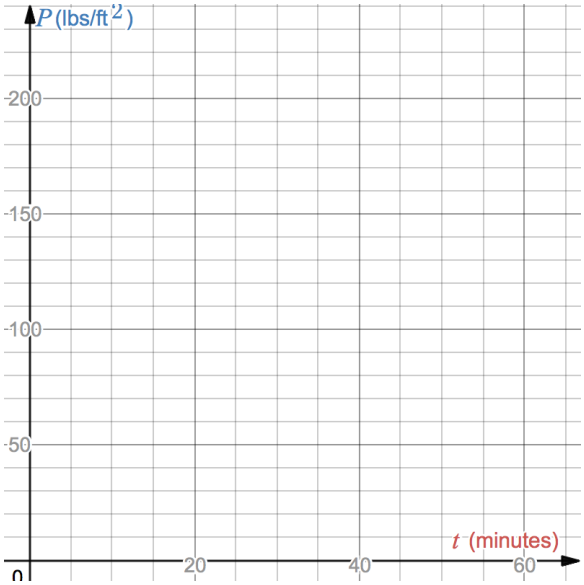
Answer (3 points): 300°.

6. (Points: 12) The pressure, P (in lbs/ft^2), in a pipe varies over time. Three times an hour, the pressure oscillates from a low of 90 to a high of 230 and then back to a low of 90. The pressure at $t = 0$ is 90.

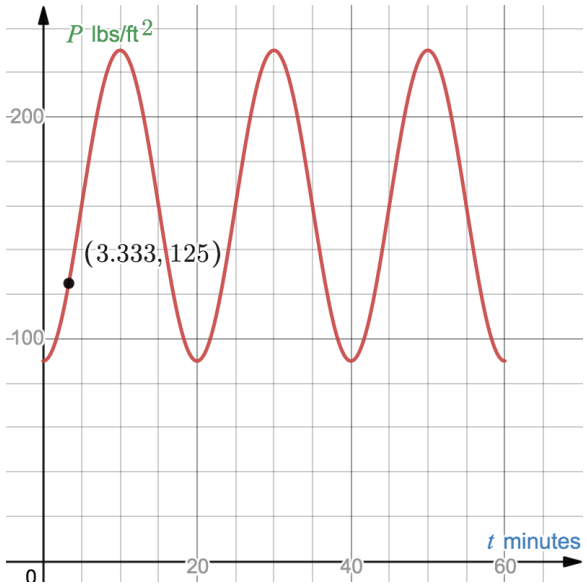
(a) Graph $P = f(t)$, where t is time in minutes.

(b) Find a possible formula for $P = f(t)$.

(c) Using your graph from part (a) $P = f(t)$ for $0 \leq t \leq 20$, estimate when the pressure first equals 125 lbs/ft^2.



Answer (4 points): (a)



Answer (4 points): (b) $P = -70\cos(\frac{\pi}{10}t) + 160$.

Answer (4 points): (c) $t = 3.333$ minutes.

7. (Points: 8) If $\cos(\alpha) = -\sqrt{3}/5$ and α is in the third quadrant,

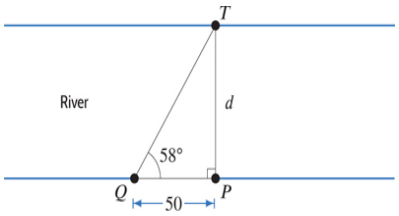
   (a) find the exact value for $\sin(\alpha)$,

   Answer (4 points): (a) $\sin(\alpha) = -\frac{\sqrt{22}}{5}$.

   (b) find the exact value for $\tan(\alpha)$.

   Answer (4 points): (b) $\tan(\alpha) = \sqrt{\frac{22}{3}}$.

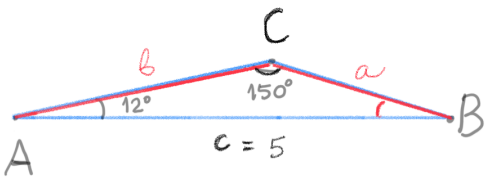
8. (Points: 6) A surveyor must measure the distance between the two banks of a straight river. She sights a tree at point T on the opposite bank of the river and drives a stake into the ground (at point P) directly across from the tree. Then she walks 50 meters upstream and places a stake at point Q. She measures angle PQT and finds that it is 58°. Find the width of the river.



   Answer (6 points): $d = 50 \cdot \tan(58°) = 80.017$ meters.

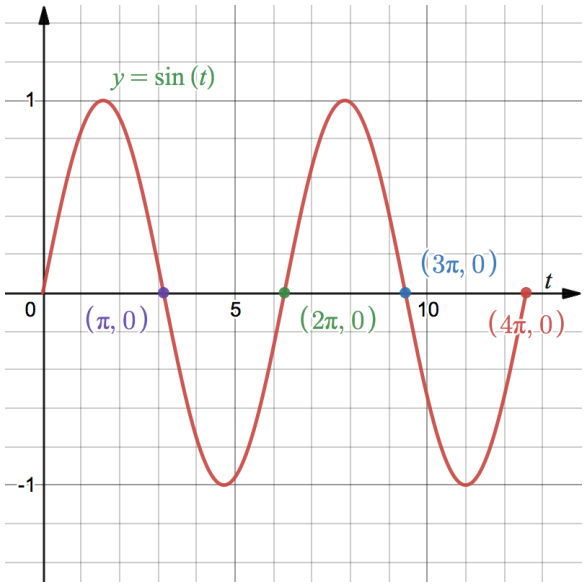
9. (Points: 8) Find the missing sides, a, b, and angle B.

$A = 12°,\ C = 150°,\ c = 5.$



Answer: (2 points): $B = 18°$; (3 points): $CB = 2.079$; (3 points): $AC = 3.090$.

10. (Points: 8) Use the graph to approximate all solutions to the equation $\sin(t) = \sqrt{2}/2$ on $0 \leq t \leq 4\pi$.



Answer: (2 points each): $\pi/4$, $3\pi/4$, $9\pi/4$, $11\pi/4$.

11. (Points: 5) Decompose the function

$$f(x) = 5\sqrt{x+3}$$

into a composition of two new functions u and v, where v is the inside function, that is $f(x) = u(v(x))$, so that $u(x) \neq x$ and $v(x) \neq x$.

Answer: (5 points any correct combination):

$v(x) = x + 3$ and $u(x) = 5\sqrt{x}$ or

$v(x) = \sqrt{x+3}$ and $u(x) = 5x$.

12. (Points: 12) Let $P = f(t) = 37.8(1.044)^t$ be the population of a town (in thousands) in year t.

(a) Evaluate $f(50)$. Describe in words what this quantity tells you.

Answer: (4 points): The population size is 325.474 thousand people at $t = 50$ years.

(b) Find a formula for $f^{-1}(P)$ in terms of P.

Answer: (4 points): $f^{-1}(P) = \frac{\ln(P) - \ln(37.8)}{\ln(1.044)}$.

(c) Evaluate $f^{-1}(50)$. Describe in words what this quantity tells you.

Answer: (4 points): The population reaches 50 thousand people in 6.496 years.